INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, 2022-23
Statistics - III, Semesteral Examination, May 4, 2023
Time: 3 Hours **Total Marks: 50**

You may use any of the results stated and discussed in class, by stating them explicitly. Calculators/Smart phones are not permitted.

1. Suppose Y is the response and X_1 and X_2 are two regressors in an experiment. It is assumed that they all have the same variance σ^2 and $Cov(Y, X_1) = \rho = Cov(Y, X_2)$, $Cov(X_1, X_2) = 0$.
(a) For what values of σ^2 and ρ can we find such a joint probability distribution?
(b) Find the multiple correlation coefficient between Y and (X_1, X_2).
(c) Let $Z = X_1 + X_2$ and $T = X_1$. Find the partial correlation coefficient, $\rho_{yz.t}$, between Y and Z given T. [4+5+5]

2. Consider the model $\mathbf{Y} = X\beta + \epsilon$, where $X_{n\times p}$ has rank $r = 4 < p < n$; also $\epsilon \sim N_n(\mathbf{0}, \sigma^2 I_n)$. Let $\hat{\beta}$ be the least squares solution of β. Suppose $\beta_j - \beta_{j+1}$ are estimable for $j = 1, 2, 3, 4$ in this model. Is it possible to test the hypothesis $H_0 : \beta_2 = (\beta_1 + \beta_3)/2$ versus $H_1 : \beta_2 \neq (\beta_1 + \beta_3)/2$? Justify your answer. If it is possible, provide the procedure to test at the 5% significance level. [10]

3. Consider the one-way model:

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij},\ 1 \leq j \leq 10;\ 1 \leq i \leq 4,$$

where ϵ_{ij} are i.i.d. $N(0, \sigma^2)$, with the standard identifiability constraints on α_i.
(a) Show that $\alpha_1 - \alpha_2$ is estimable.
(b) Compare the Scheffe and Bonferroni versions of $100(1-\alpha)\%$ simultaneous confidence intervals for $(\alpha_1 - \alpha_2, \alpha_2 - \alpha_3, \alpha_3 - \alpha_4, \alpha_4 - \alpha_1)$. [1+8]

4. (a) What is a Quantile-Quantile (Q-Q) plot? Why is it useful in regression analysis?
(b) Explain the role of the following concepts in design of experiments:
(i) randomization, (ii) replication, (iii) blocking. [4+6]

5. What is logistic regression? Compare it with linear regression [7]